PROPAGATION OF SHOCK WAVES IN NONUNIFORM PLASMA

V. V. Zakaidakov and V. S. Synakh

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Results of numerical simulation of the propagation of one-dimensional magnetohydrodynamic shock waves in a nonuniform plasma containing a magnetic field are discussed. Possible uses for the production of high velocities and temperatures and astrophysical applications are considered. The essential effect of the magnetic field is shown; acceleration of a shock wave is intensified in a medium with decreasing density.

The propagation of a magnetohydrodynamic (MHD) shock wave in a nonuniform plasma containing a magnetic field was investigated by means of numerical simulation. This is of interest in the achievement of high plasma velocities and temperatures in experimental devices, in the study of shock-wave propagation in the ionosphere, in astrophysical applications, etc. As in the propagation of a hydrodynamic shock wave in a medium with decreasing density [1-3], the velocity of an MHD shock wave sometimes increases also. It is shown here that this effect is intensified when a magnetic field is present in a nonuniform plasma. The propagation of a hydrodynamic shock wave in a medium with various laws for the variation of density has been discussed in detail [2]. The same problem involving a magnetic field has been discussed [4] but, in contrast to the present paper, under special laws for the variation of density ρ, pressure p, and magnetic field H which allowed a self-similar solution.

The instability of an MHD shock front leads to subdivision of the front into segments with a length of the order of the characteristic dimension of a plasma nonuniformity. As shown in [5], however, if the time for growth of the instability is less than the time for diffusion of the segments of the front to the system boundaries, it is impossible to take account of the instability of an MHD shock wave.

We consider the case of layer symmetry where all quantities depend only on the single spatial variable x. Let a shock wave begin to propagate at a velocity U_0 in a plasma at rest ($v = 0$; the case $v \neq 0$ will be considered separately) through the effect of a moving gas or a piston. A magnetic field is everywhere directed parallel to the MHD shock front. The plasma is assumed ideal both ahead of the front and behind it (conductivity $\sigma = \infty$). This is not an idealization in the majority of cases. Thus, in the atmosphere of a star, $\sigma \geq 10^{14}$-10^{16} $\sec^{-1}$, which corresponds to a magnetic Reynolds number $Re_m \geq 10^5$. For experimental devices of typical size (~1 m), $Re_m \sim 100$; for the ionosphere at ~300 km with conductivity $\sigma \sim 10^{10}$-10^{11} $\sec^{-1}$ and typical large-scale phenomena (~1000 km), $Re_m \sim 10^2$-10^4, so that dissipative terms in the equations can be neglected. The laws for the variation of ρ_0, p_0, and H_0 in the following material are selected to be as close as possible to those for an actual situation.

We start from the usual MHD equations for the motion of a nonviscous, non-heat-conducting ideal plasma:

$$\partial\rho/\partial t + \partial\rho v/\partial x = 0;$$
$$\partial H/\partial t + \partial Hv/\partial x = 0;$$
$$\rho\partial v/\partial t + \rho v\partial v/\partial x + \partial(p + H^2/8\pi)/\partial x = 0;$$
$$(\partial/\partial t + v\partial/\partial x)S = 0; \quad S = S(p, \rho),$$

where v is the velocity and S is the entropy. Assuming that the plasma ahead of the MHD shock front is at rest, we write relations at the discontinuity which are solved relative to quantities behind the front,

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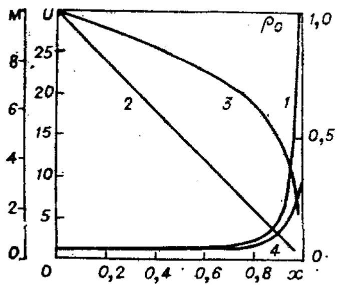

Fig. 1

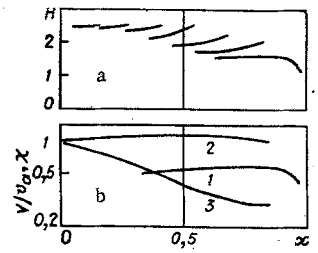

Fig. 2

$$v_1 = U(1 - a/2);$$
$$\rho_1 = 2\rho_0/a;\ H_1 = 2H_0/a;$$
$$p_1 = \frac{1}{2}\rho_0 U^2\left[\frac{\gamma+1}{\gamma(1+\alpha)} - a\right] - \frac{1}{2\pi}\left(\frac{H_0}{a}\right)^2,$$

where

$$\alpha = \frac{\gamma-1}{\gamma+1}\left[1 + \frac{2\gamma}{\gamma-1}\frac{p_0 + \frac{H_0^2}{8\pi}}{\rho_0 U^2}\right];$$
$$a = \alpha + \sqrt{\alpha^2 + \frac{2-\gamma}{\gamma+1}\frac{H_0^2}{\pi\rho_0 U^2}}.$$

The subscript 0 denotes quantities ahead of the front and the subscript 1, quantities behind the front; U is the velocity of the MHD shock wave; γ is the adiabatic index.

For the numerical solution, a modified method of characteristics (see [6], for example) was used in which Lagrangian coordinates were not employed and iteration procedures were eliminated. As theoretical and numerical studies have shown, the scheme is stable and yields smooth solutions of second-order accuracy. The shock wave is considered explicitly. The coordinates of the front, the velocity of the shock wave, and quantities behind the shock front were calculated by the discretization of boundary conditions described in [7]. In the following, we use the dimensionless variables $x = x/L$; $v = v/U_0$; $t = tL/U_0$;

$$\rho = \rho/\rho_0(0);\ p = p/\rho_0(0)U_0^2;\ H = H/\rho_0(0)^{1/2}U_0,$$

where L is a characteristic dimension and $\rho_0(0)$ is the density ahead of the MHD shock front at zero time.

The calculations were made for the following three typical cases.

1. Linear variation of density. The plasma ahead of the front is at equilibrium ($v_0 = 0$).

2. The case of a uniform and equilibrium plasma, $\partial P/\partial x = 0$, where $P = p + H^2/8\pi$ is the total pressure, but $\partial p/\partial x \neq 0$. The magnetic field and pressure vary smoothly enough so that a reflected shock wave is not produced. There is a definite interest in the case where the length δ over which H and p change significantly is much less than the total distance L traversed by the MHD shock wave, $\delta \ll L$.

3. The creation of density gradients by the expansion of gas into a vacuum makes it possible to obtain density distributions $\rho_0 \sim x^3$ and $\rho_0 \sim x^5$ for monatomic and diatomic gases (collision of MHD shock wave and rarefaction wave). In the calculation of this case, it is considered that the plasma ahead of the shock front is not at equilibrium ($v_0 \neq 0$) and the relations at the discontinuity are appropriately changed by the inclusion of a nonzero plasma velocity ahead of the front.

The calculations revealed that the nature of the flow behind an MHD shock front was qualitatively identical for all initial density distributions considered.

§1. We consider the case of a linear variation in density, $\rho \sim x$. The dependence of the MHD shock-wave velocity on the coordinate x is shown in Fig. 1 (curve 1). At zero time, the parameter $\beta = 8\pi p/H^2$ (ratio between gas-kinetic pressure and magnetic pressure) was taken to be $\beta_0 = 0.1$ and the Mach number $M = U/c^*$ was assumed to be 10, where

$$c^* = \left(\frac{\gamma p_0 + \frac{H_0^2}{4\pi}}{\rho_0}\right)^{1/2}$$

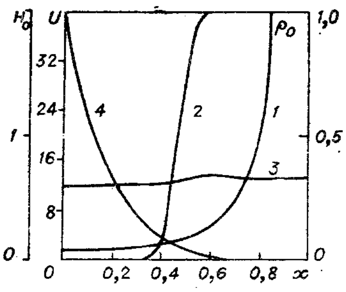

Fig. 3

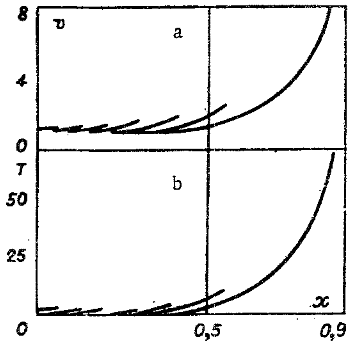

Fig. 4

is the propagation velocity for small perturbations.

The figure also shows the dependence of the velocity of a hydrodynamic shock wave on the coordinate x for a linear decrease in density (curve 4). It is clear that the velocity of an MHD shock wave grows considerably more rapidly than in the hydrodynamic case.

Note that the effective velocity of sound, c^*, ahead of the front increases more rapidly than the velocity of the MHD shock wave, and the Mach number decreases. Figure 1 also shows the dependence of M on the coordinate of the MHD shock front (curve 3) and the dependence $\rho_0(x)$ for the unperturbed density (curve 2).

The nature of magnetic field variation behind the front depends significantly on the values of M and β. Profiles of H at various times for the case under discussion are shown in Fig. 2a. As is clear, the magnetic field increases with increase in coordinate at early times and is maximal immediately behind the MHD shock front; it falls with increasing x $(\partial H/\partial x < 0)$ at later times when the Mach number is falling. The changes in the behavior of the quantity

$$\chi = v_a/(U - v_1)$$

correspond to these two modes of MHD shock-wave propagation, where $v_a = H_0/\sqrt{4\pi\rho_0}$ is the Alfvén velocity ahead of the shock front; $U - v_1$ is the MHD shock-wave velocity relative to the plasma behind the front. The relation $\chi = \chi(x)$ is shown in Fig. 2b (curve 1 for case 1 and curve 2 for case 3). Note that the quantity χ is the analog of the coefficient of proportionality between v_a and U for the self-similar solution [3]. Shown for comparison is the dependence of the ratio between the velocity of the MHD shock wave and the Alfvén velocity ahead of the front (curve 3) which indicates that in case 1 acceleration occurs more slowly than in the self-similar solution [3].

§2. The dependence on coordinate of the unperturbed magnetic field H_0 (curve 2) and of the velocity of the MHD shock wave (curve 3) is shown in Fig. 3 for an equilibrium plasma of uniform density. The following initial values were used: $M = 100$, $\beta_0 = 10^4$, and $\gamma = 5/3$. As indicated by the results of the numerical calculations, the change in MHD shock-wave velocity is small, which demonstrates the very strong influence of the magnetic field with a gradient in the density ρ_0. The sign of the acceleration of the shock front is determined by the value of adiabatic index. Thus, for $\gamma = 2.5$ an MHD shock wave is decelerated under the same initial conditions. The case $\gamma = 2$ is the limiting case.

Thus, the existence of a gradient in the magnetic field alone does not lead to significant effects.

§3. The dependence on coordinate of the velocity of an MHD shock wave and of the density ρ_0 at the shock front for case 3 is shown in Fig. 3 (curves 3 and 4, respectively). A comparison with the case of linear density variation shows that the large density gradient does not lead to significant differences, but one must note the more abrupt and rapid occurrence of all processes. Curve 2 in Fig. 2b shows the behavior of the quantity χ as a function of the coordinate of the front for this case. Profiles of the temperature T and of the plasma velocity v behind the MHD shock front at various times are shown in Fig. 4a, b. Note the considerable rise in temperature behind the shock front. Since the density-variation law $\rho \sim x^5$ is obtained fairly simply under experimental conditions, this case is promising for the production of high plasma temperatures and velocities behind an MHD shock front.

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